

國立彰化師範大學 99 學年度碩士班招生考試試題

系所：數學系

組別：甲組

科目：機率與統計

☆☆請在答案紙上作答☆☆

共 1 頁，第 1 頁

- (20%) A group of 10 people consisting of 5 men and 5 women is randomly arranged into 5 pairs of 2 each. Let X denote the number of pairs that consist of a man and a woman. Compute $E(X)$ and $Var(X)$.
- (30%) Let X and Y be two Poisson distributed random variables with $E(X) = \lambda$, $E(Y) = \mu$.
 - (15%) If X and Y are independent, find the conditional distribution of X given that $X + Y = n$, where n is a nonnegative integer.
 - (15%) If $\lambda > \mu$, and there exists a random variable Z such that Y and Z are independent and $X = Y + Z$, find the distribution of Z .
- (20%) Suppose X and Y are independent random variables with X being uniform on $(0, 1)$ and Y being exponential with mean 1. Let $U = XY$ and $V = X + Y$. Compute the following:
 - (4%) $E[U]$ and $E[V]$.
 - (4%) The covariance of U and V .
 - (4%) The conditional density $f(u|y)$ of U given $Y = y$.
 - (8%) The probability density function for V .
- (30%) Let X equal the forced vital capacity (FVC) in liters for a college student. (The FVC is the amount of air that a student can force out of his/her lungs.) Assume that the distribution of X is approximately $N(\mu, \sigma^2)$. Suppose it is known that $\mu = 3.4$ liters. A baseball coach claims that the FVC of baseball players is greater than 3.4. He plans to test his claim with a random sample of size $n = 9$.
 - (2%) Define the null hypothesis.
 - (2%) Define the alternative (coach's) hypothesis.
 - (4%) Define the test statistic.
 - (4%) Define the critical region(s) for which $\alpha = 0.05$. Draw a figure illustrating your critical region(s).
 - (8%) Calculate the value of the test statistic given that the random sample yielded the following FVC's:
3.4 3.6 3.8 3.3 3.4 3.5 3.7 3.6 3.7
 - (2%) What is your conclusion?
 - (2%) What is the approximate p -value of this test?
 - (6%) Find the power function $K(\mu)$ for this test.