國立彰化師範大學 97 學年度碩士班招生考試試題

系所：數學系碩士班　　組別：甲組　　科目：機率與統計

☆☆請在答案紙上作答☆☆　　

請回答下列各題

1. Please define a t-distributed random variable with degree of freedom r by expressing it as a function of a standard normal random variable and a chi-square random variable.（10 points）

2. Consider the function $f(x,y)=\begin{cases} \frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}} & , for\ (x,y) outside [-1,1]\times[-1,1] \\ \frac{1}{2\pi}e^{-\frac{x^2+y^2}{2}}+\frac{1}{2\pi e}x\,y^3 & , for\ (x,y) in [-1,1]\times[-1,1] \end{cases}$。

   Show that $f(x,y)$ is a non-bivariate normal p.d.f. but their marginals are N(0,1) distributed. (15 points)

3. Let X and Y be two random variables with densities

   $$f_X(x)=\frac{x^{p-1}e^{-x/\lambda}}{\Gamma(p)\lambda^p},\quad f_Y(y)=\frac{y^{q-1}e^{-x/\lambda}}{\Gamma(q)\lambda^q},\quad x,y>0,$$ respectively, for some $\lambda>0$。 Please find the probability density function of $U=X/(X+Y)$. What is the type of distribution for U? (20 points)

4. A new technology is applied to 13 students to enhance their ability of arithmetic. The following scores are recorded before and after the examination.

Before	：	6.3	4.4	4.2	3.9	3.2	2.1	3.1	2.5	1.9	2.3	0.8	5.5	2.5
After	：	6.6	4.3	4.5	3.2	3.5	3.0	2.9	3.2	2.3	2.1	1.1	4.6	3.7

   If the significance level is assumed to be α=0.05,

   (a) How to construct the hypothesis ? (5 points)

   (b) Please indicate the testing statistics and illustrate the method of testing.(e.g. chi-square testing or t-testing,…) You only have to show your strategies but the calculations. (10 points)

   (c) Before (b), what else assumptions should we made on the samples? (5 points)

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5. A new technology is applied to class A with 13 pupils (小孩) to enhance(加強)their ability of arithmetic. The pupils of class B are instructed without the new technology. The following are the scores of the two classes.

Class B：	6.3	4.4	4.2	3.9	3.2	2.1	3.1	2.5	1.9	2.3	0.8	5.5	2.5
Class A：	6.6	4.3	4.5	3.2	3.5	3.0	2.9	3.2	2.3	2.1	1.1	4.6	3.7

If the significance level is assumed to be α=0.05,

(a) How to construct the hypothesis？(5 points)

(b) Please indicate the testing statistics and illustrate（說明）the method of testing.(e.g. chi-square testing or t-testing,…) You only have to show your strategies (策略) but the calculations. (10 points)

(c) Before (b), what else assumptions should we made on the samples? (5 points)

6. Let $X_1,...,X_n$ be i.i.d. random variables with Weibull distribution, i.e. their marginal density is

$$f(x,\beta)=\frac{\gamma}{\beta}x^{\gamma-1}\exp(-x^{\gamma}/\beta),\quad x>0,\ \text{for some}\ \gamma>0.$$

Please find the MLE (maximum likelihood estimate) of β. (15 points)