國立彰化師範大學 98 學年度碩士班招生考試試題

系所：數學系　　　　組別：甲、乙、丙、丁組　　　　科目：線性代數

☆☆請在答案紙上作答☆☆　　　　

1. Let $T: R^3 \to R^3$ be the projection of the space R^3 on the plane $x+2y+3z=0$. Find the eigenvalues of T and find a basis of the corresponding eigenspaces. (12%)

2. Find all possible Jordan forms and their corresponding minimal polynomials for a 6×6 matrix over complex numbers with characteristic polynomial $(x-1)^3(x-2)^1(x-3)^2$. (12%)

3. Let $A=\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. Find an orthogonal matrix P and a diagonal matrix D such that $P^T AP = D$, where P^T is the transpose of P. (20%)

4. Let A and B be two complex $n\times n$ matrices. Show that AB and BA have at least one eigenvalue in common. That is, there is a complex number λ such that λ is not only an eigenvalue of AB but also an eigenvalue of BA. (12%)

5. (a) Let A be a real $n\times n$ matrix. Show that if $A^k=0$ for some integer $k\ge 0$, then $A^n=0$ (7%)
   (b) Let A be a real $n\times n$ matrix. Show that if A is invertible, then A^{-1} can be written as a linear combination of $I_n, A, A^2, ..., A^{n-1}$, where I_n denotes the identity matrix. (8%)

6. Let V be a finite-dimensional real vector space and $T: V \to V$ a linear transformation. Let $R(T)$ denote the range space of T and $N(T)$ the null space of T. Prove that the dimension of $N(T)$ plus the dimension of $R(T)$ equals the dimension of V. That is, $\dim N(T) + \dim R(T) = \dim V$. (15%)

7. Let V be a finite-dimensional real vector space with an inner product $\langle , \rangle$ and let W be a subspace of V. Define $W^\perp = \{\vec{v}\in V \mid \langle \vec{v}, \vec{w}\rangle = 0, \forall \vec{w}\in W\}$, the orthogonal complement of W.
   (a) Show that $W^\perp$ is a subspace of V. (2%)
   (b) Show that $V = W + W^\perp$. (7%)
   (c) Show that $(W^\perp)^\perp = W$. (5%)