國立彰化師範大學 99 學年度碩士班招生考試試題

系所：數學系　　　組別：甲、乙、丙組　　　科目：線性代數

☆☆請在答案紙上作答☆☆　　　

1. Let $V=\{ae^{x}+be^{2x}+ce^{3x}+de^{4x} \mid a,b,c,d\in\mathbb{R}\}$. Show that $\{e^{x},e^{2x},e^{3x},e^{4x}\}$ is a basis of V. (14%)

2. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be given by

$$T(x,y,z)=(x-y+3z,5x+6y-4z,7x+4y+2z).$$

Find $\dim T(\mathbb{R}^3)$ and $\dim\ker(T)$. (14%)

3. Let $A=\begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$. Find all eigenvalues and eigenvectors of A. Explain why A is diagonalizable and find a matrix that diagonalizes A. (18%)

4. Let $f(u,v)$ be a real valued function on $\mathbb{R}^2\times\mathbb{R}^2$ such that

(a) for fixed u, it is a linear function on v, and, for fixed v, it is a linear function on u;

(b) $f(v,u)=-f(u,v)$.

Show that $f(u,u)=0$ and then deduce that $f(u,v)$ is a multiple of the determinant of the 2×2 matrix $[u\ v]$ with column vectors u, v. (18%)

5. (a) Find a 3×3 orthogonal matrix U that maps the x-y plane $z=0$ to the plane $P:x+y+z=0$.

(b) Use U as the matrix of change of coordinates to deduce the formula for the rotation around the axis $L=\{(t,t,t)\mid t\in\mathbb{R}\}$ with rotation angle 90° (counterclockwise). (18%)

6. Suppose $A\neq I$ is a 3×3 real matrix such that $A^3=A^2-A+I$.

(a) Find all possible eigenvalues of A.

(b) Determine the minimal and characteristic polynomial of A.

(c) Is A diagonalizable? Explain your answer. (18%)