# 國立彰化師範大學103學年度碩士班招生考試試題

系所： 數學系　　組別： 乙組　　科目： 高等微積分

☆☆請在答案紙上作答☆☆　　

**1.** Define real function $f$ on $R^2$ by

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^4} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}.$$

Prove that $f$ is bounded on $R^2$ and that $f$ is not continuous at $(0,0)$. (20%)

**2.** If $L: R^p \to R^q\,(p,q\in N)$ be a linear transformation, then prove that $L$ is uniformly continuous on $R^p$. (20%)

**3.** Let $f_n(x)=\dfrac{1-e^{-nx}}{\sqrt{x}}$ *on* $(0,\infty)$. Determine $f(x)=\lim\limits_{n\to\infty} f_n(x)$ *for* $0<x<\infty$, and show that $f_n$ converges uniformly to $f$ on $[a,\infty)\,(0<a<\infty)$, but not on $(0,\infty)$. (20%)

**4.** Let $f(y)$ be a continuous function on $[a,b]$, and let $F(x)=\int_a^b f(y)\,|x-y|\,dy$.

Show that $F(x)$ is differentiable, and find $F'(x)$. (20%)

**5.** Does the improper integral $\displaystyle\int_0^\infty \frac{1+\sqrt{x}}{\sqrt{x+x^2}}\,dx$ converge? Prove your answer. (20%)