# 國立彰化師範大學 99 學年度碩士班招生考試試題

系所：數學系　　組別：乙組　　科目：高等微積分

☆☆請在答案紙上作答☆☆　　

1. (20%) Use the $\varepsilon - \delta$ definition of limit to show $\lim_{x\to 0}\sqrt{5x+7} = \sqrt{7}$.

2. (15%) Let $f$ be a function from $\mathbb{R}$ into $\mathbb{R}$. Suppose that $|f(x)-f(y)| \le 2|x-y|^{1.01}$ for $x, y \in \mathbb{R}$. Show that $f$ is a constant function on $\mathbb{R}$.

3. (25%) Let $f : [a,b] \to \mathbb{R}$ be differentiable on $[a,b]$. Show that $f'$ is continuous on $[a,b]$ if and only if for every $\varepsilon > 0$ there is a $\delta(\varepsilon) > 0$ such that if $0 < |x-y| < \delta(\varepsilon)$ and $x, y \in [a,b]$, then

$$\left|\frac{f(x)-f(y)}{x-y} - f'(x)\right| < \varepsilon.$$

4. (15%) Show the series $\sum_{n=1}^{\infty} \frac{tan^{-1}(x^n)}{n^{1.03}}$ define a continuous function on $\mathbb{R}$.

5. (25%) Let $\{f_n\}$ be a sequence of continuous functions from $[0,1]$ to $\mathbb{R}$. Suppose that $f_n(x) \to 0$ as $n \to \infty$ for each $x \in [0,1]$ and also that, for some constant $K > 0$ we have

$$\left|\int_0^1 f_n(x)dx\right| \le K \quad \text{for all } n \in \mathbb{N}.$$

Does $\lim_{n\to\infty}\int_0^1 f_n(x)dx = 0$?
Prove or disprove it.