國立彰化師範大學 98 學年度碩士班招生考試試題

系所：數學系　　　　組別：乙組　　　　科目：高等微積分

☆☆請在答案紙上作答☆☆　　　　

1. (a) State Bolzano-Weierstrass Theorem for R^n.
   (b) Suppose $E \subset R^n$ is a compact set, and $f : E \to R$ is a continuous function.
   Show that there exists $c \in E$, such that $f(c) \geq f(x)$, for all $x \in E$. (20%)

2. Suppose that $f : R \to R$ is a continuous function, and $f(f(a)) = a$ for some $a \in R$.
   Show that there exists $c \in R$ such that $f(c) = c$. Hint: Use Intermediate Value Theorem.
   (20%)

3. Assume that $f_n \to f$ uniformly on $[a,b]$ and suppose that there is a constant $M > 0$ such that $|f_n(x)| \leq M$ for all $x \in [a,b]$ and all n.
   Define : $h_n(x) = \sin(f_n(x)), h(x) = \sin(f(x))$, for $x \in [a,b]$
   Prove that $h_n \to h$ uniformly on $[a,b]$. (16%)

4. Give an approximate value of the integral $\int_0^1 \sin(x^2)dx$, and prove that its error is less than 10^{-3}. (12%)

5. Evaluate following integrals: (16%)

$$\iiint_{\mathbf{R}^3} e^{-2(x^2+y^2+z^2+xy+yz+zx)}\,dxdydz$$

6. Define $f(x,y) = x^2 - (y-1)^2$, $E = \{(x,y) \mid y \geq 0, x^2 + y^2 \leq 4\}$. Find the absolute maximum and minimum of f on E. (16%)