國立彰化師範大學 97 學年度碩士班招生考試試題

系所：數學系碩士班　　組別：乙組　　科目：高等微積分

☆☆請在答案紙上作答☆☆　　共 1 頁，第 1 頁

請回答下列各題（配分如題所示，共 100 分）

1. (23%)

   Assume $f(x)=\begin{cases} 1 & \text{if} \quad 0\le x\le 1; \\ -1 & \text{if} \quad 1<x\le 2. \end{cases}$

   Prove that f is Riemann integrable on [0, 2].

2. (23%)

   Suppose f is Riemann integrable on [0, 1].

   Prove that $\int_0^1 x^k f(x)dm \to 0$ as $k\to\infty$.

3. (23%)

   Use Lagrange Multipliers to find the minimum value of

   $f(x,y,z)=x^2+y^2+z^2$

   subject to the constraints

   $x+y-3=0$

   $x+z-5=0$.

4. (23%)

   Evaluate $\int_{-\infty}^{\infty} e^{-x^2}dx$ by any method that you know.

5. (8%)

   ϕ is said to be a convex function on $(a,\ b)$ if

   $\phi(\alpha x+(1-\alpha)y)\le \alpha\phi(x)+(1-\alpha)\phi(y)$

   for all $0\le\alpha\le 1$ and $x,y\in(a,\ b)$.

   If ϕ be convex on $(a,\ b)$, prove that ϕ is continuous on (a,b).