

國立彰化師範大學 98 學年度碩士班招生考試試題

系所：數學系

組別：丁組

科目：微積分

☆☆請在答案紙上作答☆☆

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1. Define $f(x, y) = x^2 - (y-1)^2$, $E = \{(x, y) | y \geq 0, x^2 + y^2 \leq 4\}$. Find the absolute maximum and minimum of f on E . (15%)

2. Find the following indefinite integral: (15%)

$$\int \frac{dx}{x^3 - x^2 + 2x - 2}$$

3. Let $f(x) = x^2 e^{-\frac{x}{2}}$, $x \in (-\infty, \infty)$. Answer following questions: (25%)

(a) At which value of x does f have local minimum or local maximum?

(b) Does f have absolute maximum or minimum on $(-\infty, \infty)$?

(c) At which interval is f increasing or decreasing?

(d) At which interval is f concave-up or concave-down?

(e) Sketch the graph of f .

(f) Find the definite integral $\int_0^{\infty} f(x) dx$

4. Evaluate following integrals: (15%)

(a) $\iint_{R^2} e^{-(x-1)^2 - \frac{y^2}{3}} dx dy$

(b) $\iint_{R^2} xy^2 e^{-(x-1)^2 - \frac{y^2}{3}} dx dy$

5. Evaluate following limits: (15%)

(a) $\lim_{x \rightarrow 0} \frac{\ln(1+x^2) - \sin(x^2)}{x^4}$ (b) $\lim_{x \rightarrow 0^+} \frac{e^{-x}}{x^2}$

6. Determine whether the following series converge, and explain why. (15%)

(a) $\sum_{n=1}^{\infty} \frac{n!}{n^n} 2^n$ (b) $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^3 + \sqrt{n}}$ (c) $\sum_{n=1}^{\infty} n e^{-\sqrt{n}}$