

國立彰化師範大學 97 學年度碩士班招生考試試題

系所：數學系碩士班

組別：丁組

科目：微積分

☆☆請在答案紙上作答☆☆

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請回答下列各題(配分如各題所示，共 100 分)

1. (16%)

Find the minimum value of $x^2 + y^2 + z^2$ subject to the constraints $x - y = 2$ and $y^2 + 2y - z = -2$.

2. (16%)

Evaluate the indefinite integral $\int \sqrt{3x^2 + 4x + 1} dx$.

3. (10%)

Evaluate the integral $\int_0^3 \int_{y/3}^1 \sin(x^2) dx dy$.

4. (24%)

Determine whether the following series converge or diverge. Give reasons for your answers.

(a) $\sum_{n=1}^{\infty} \frac{2^n n!n!}{(2n)!}$ (b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/2}}$ (c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$.

5. (12%)

Show that the function

$$f(x, y) = \begin{cases} \frac{3xy^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at $(x, y) = (0, 0)$.

6. (12%)

Use change of variables in a double integral to evaluate the integral

$$\iint_R (x + y)^2 e^{(y-x)} dy dx$$

where R is the region inside the square with vertices $(0, 0)$, $(1, 1)$, $(0, 2)$, and $(-1, 1)$.

7. (10%)

Evaluate each limits.

(a) $\lim_{x \rightarrow 0} \frac{\ln(1 + x^2)}{1 - \cos(2x)}$ (b) $\lim_{x \rightarrow 0} \frac{\sin(x) - \tan(x)}{x^3}$