

國立彰化師範大學九十六學年度碩士班招生考試試題

系所：數學系

組別：乙組

科目：高等微積分

請在答案紙上作答

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$\frac{20}{100}$ 1. Let $f(x) = \begin{cases} \sin(\frac{1}{x}) & x \in (0, 1]; \\ 0 & x = 0. \end{cases}$

Is f Riemann integrable on $[0, 1]$? Prove your conclusion.

$\frac{20}{100}$ 2. Let $f(x) = \begin{cases} x^2 & \text{if } x \text{ is irrational;} \\ 0 & \text{if } x \text{ is rational.} \end{cases}$

Is f differentiable at 0? Prove your conclusion.

$\frac{20}{100}$ 3. Let $a_n = (-1)^n(1 + \frac{1}{n})$. Find $\limsup_{n \rightarrow \infty} a_n$ and $\liminf_{n \rightarrow \infty} a_n$.

Prove your conclusion.

$\frac{20}{100}$ 4. Let $s(x) = \sum_{n=1}^{\infty} \frac{1}{n} x^n$. Find all real x such that the infinite series converges.

Prove your conclusion.

$\frac{20}{100}$ 5. Find the minimum value of

$$f(x, y, z) = x^2 + y^2 + z^2$$

subject to the constraints

$$x + y - 3 = 0 \quad \text{and} \quad x + z - 5 = 0.$$

Why does the minimum value exist? Prove your conclusion.