國立彰化師範大學103學年度碩士班招生考試試題

系所：物理學系　　組別：甲組(選考乙)　　科目：物理數學

☆☆請在答案紙上作答☆☆　　共 1 頁，第 1 頁

1. y_p is a solution of the given equation. Solve the given initial value problem. **(10%)**

$$y''-6y'+9y=2e^{3x},\quad y(0)=0,\quad y'(0)=1;\quad y_p=x^2e^{3x}$$

2. (a) Find the Laplace transforms of $e^{-t}\sin(\omega t+\theta)$. **(10%)**

(b) Find $f(t)$ if $\mathcal{L}(f)$ equals $\dfrac{s-2}{s^2-4s+5}$. **(10%)**

3. (a) Find the Fourier series of the function $f(x)$, which is assumed to have the period 2π.

$$f(x)=\begin{cases}1 & \text{if } -\pi/2<x<\pi/2\\ -1 & \text{if } \pi/2<x<3\pi/2\end{cases}$$ **(10%)**

(b) Represent the following function $f(x)$ by a Fourier sin series.

$$f(x)=L-x\qquad (0<x<L)$$ **(10%)**

4. (a) Suppose that $\mathbf{A}$ is a 3×3 matrix with eigenvalues $\lambda_1=5$, $\lambda_2=5$, and $\lambda_3=1$, and corresponding eigenvectors

$$|\mathbf{x}_1\rangle=\begin{pmatrix}-1\\1\\0\end{pmatrix},\quad |\mathbf{x}_2\rangle=\begin{pmatrix}0\\0\\1\end{pmatrix},\quad |\mathbf{x}_3\rangle=\begin{pmatrix}1\\1\\0\end{pmatrix}.$$

Find the matrix $\mathbf{A}$. **(10%)**

(b) Compute the matrix $\mathbf{A}^{20}$. **(10%)**

5. (a) Expand the complex function $f(z)=\dfrac{\cos(\pi z)}{z(z-2)^3}$ in a Laurent series valid for $0<|z-2|<2$. **(10%)**

(b) Calculate the integral $\displaystyle\int_0^{2\pi}\frac{d\theta}{1+8\cos^2\theta}$ by using residue theory. **(10%)**

6. Solve the second-order partial differential equation $\dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{\partial u}{\partial y}+2$ with $u(x,0)=0$ and $\dfrac{\partial u(0,y)}{\partial y}=y^2$. **(10%)**