國立彰化師範大學 102 學年度碩士班招生考試試題

系所：物理學系　　組別：甲組(選考乙)　　科目：物理數學

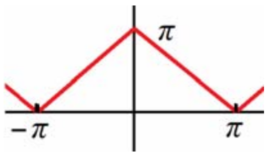
☆☆請在答案紙上作答☆☆　　共 1 頁，第 1 頁

1. Solve the initial value problem : $2xyy' = 3y^2 + x^2,\ y(1) = 2$. (10%)

2. Find $f(t)$ if the Laplace Transforms $\mathcal{L}(f)$ equals :

   (a) $\dfrac{1}{s^3 + as^2}$　　(10%)

   (b) $\ln \dfrac{s^2+1}{(s-1)^2}$　　(10%)



3. Find the Fourier series of $f(x) = \pi - |x|$, if $-\pi < x < \pi$, and $f(x+2\pi) = f(x)$, period $p = 2\pi$. (20%)

4. Consider an eigenvalue problem

$$\frac{d^2u}{dx^2} + u = -\lambda u, \quad u(0) = 0, \quad u(1) = 0.$$

   (a) Is $\lambda = 0$ an eigenvalue? Explain your answer. (5%)

   (b) Find the eigenvalues and eigenfunctions for this problem. (10%)

5. (a) Expand the complex function $f(z) = \dfrac{1}{z(1-z)^2}$ in a Laurent series valid for $0 < |z| < 1$.(5%)

   (b) Evaluate the Cauchy principal value of the integral $I = \int_{-\infty}^{\infty} \dfrac{x \sin x}{x^2+1} dx$.(10%)

6. If two matrices can be written as

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } \mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix},$$

   satisfying the similarity transformation $\mathbf{A} = \mathbf{RDR}^{-1}$, then for $\lambda_1 \le \lambda_2 \le \lambda_3$.

   (a) What are the values of λ_1, λ_2 and λ_3? Also find their corresponding eigenvectors. (10%)

   (b) Find the matrix $\mathbf{R}^{-1}$.(10%)