

國立彰化師範大學 99 學年度碩士班招生考試試題

系所：物理學系

組別：甲組

科目：物理數學

☆☆請在答案紙上作答☆☆

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1. (15%) The angular momentum \mathbf{L} and angular velocity $\boldsymbol{\omega}$ for a rigid body is related by the equation $L_i = I_{ij}\omega_j$ where I_{ij} is the inertia tensor and $i, j = 1, 2, 3$ denote the Cartesian components. Suppose that the inertia tensor of a rigid body is given by

$$I_{ij} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Find a direction of $\boldsymbol{\omega}$ such that it is parallel to \mathbf{L} .

2. (15%) Consider the vibration of the carbon dioxide CO_2 which is a linear molecule $O-C-O$. Let m and M be the masses of C and O , respectively. The interaction between C and O can be approximated as a harmonic potential with the force constant k . Determine the eigenfrequencies of this system.
3. (20%) Evaluate the following integrals:

(a) $P \int_{-\infty}^{\infty} dx \frac{x}{x^3 + 1}$

(b) $\int_0^{2\pi} d\theta \frac{1}{1 + \sin^2 \theta}$

4. (20%) Find the general solution of the following differential equations:

(a) $\frac{d^2 y}{dx^2} + y = 2xe^x$

(b) $4x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + (x^2 - 1)y = 0$

5. (20%) A vector field is defined by the equation $\vec{A} = x^2 y \hat{x} - xy^2 \hat{y}$.

(a) Calculate $\nabla \cdot \vec{A}$.

(b) Calculate $\nabla \times \vec{A}$.

(c) Calculate $\nabla^2 \vec{A}$.

(d) Along the straight line, calculate the integral $\int_{(1,1)}^{(4,2)} \vec{A} \cdot d\vec{r}$.

6. (10%) Let $f(t) = t \sin t$, find the Laplace transformation of $f(t)$, given $f(0) = 0$.