國立彰化師範大學 97 學年度碩士班招生考試試題

系所：物理學系碩士班　　科目：物理數學

☆☆請在答案紙上作答☆☆　　

請回答下列各題(配分如各題所示，共 100 分)

1. A force $\vec{F}$ is given by the formula $\vec{F} = r^2\hat{r} + z\hat{x} - x\hat{z}$, where *x*, *y*, *z* are Cartesian coordinates (in the usual right-handed system in which $\hat{x} \times \hat{y} = \hat{z}$), with $\hat{x}, \hat{y}, \hat{z}$ unit vectors in the *x*, *y*, and *z* directions; $\vec{r}$ is the vector from the origin to the point (*x*, *y*, *z*), r is its magnitude, and $\hat{r}$ is a unit vector in the direction of $\vec{r}$.
   (a) (5%) Write the vector $\vec{F}$ in terms of $\hat{x}, \hat{y}, \hat{z}$ unit vectors.
   (b) (5%) Calculate $\vec{\nabla} \times \vec{F}$.
   (c) (10%) Calculate the line integral $\oint \vec{F} \cdot d\vec{\rho}$, where the path is a unit circle in the *xz* plane, centered at $(x, y, z) = (0.5, 0, 1.5)$, with the path direction such that, starting at $(x, y, z) = (1.5, 0, 1.5)$, the path passes, in order, $(0.5, 0, 2.5)$, $(-0.5, 0, 1.5)$, $(0.5, 0, 0.5)$, ending upon returning to $(1.5, 0, 1.5)$.

2. A uniform bar of length *L* is clamped at $x = 0$ and $x = L$ such that it lies in a horizontal plane. Provided that the deviations from equilibrium are small, the vertical vibration $u(x, t)$ satisfies the fourth-order equation

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0.$$

   (a) (10%) Looking for a separable solution of the form $u(x, t) = X(x) \times T(t)$, find the ordinary differential equations satisfied by $X(x)$ and $T(t)$.
   (b) (10%) Find the most general solution $u(x, t)$ which satisfies the boundary conditions:

$$u(0, t) = 0,\ u(L, t) = 0,\ \left.\frac{\partial^2 u(x,t)}{\partial x^2}\right|_{x=0} = 0,\ \left.\frac{\partial^2 u(x,t)}{\partial x^2}\right|_{x=L} = 0, \text{ and } u(x,0) = 0.$$

3. (10%) Consider the periodic function (with period 2π) defined as

$$f(x) = \begin{cases} \frac{1}{2}\sin x, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}.$$

   If the Fourier series of such a function can be expressed as

$$f(x) = A\sin x + B + \sum_{n=1}^{\infty} \frac{C_n}{\pi}\cos(2nx),$$

   what are the constants A, B and C_n?

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4. (8%) Let the components of the inertia tensor in a Cartesian system be represented by

$$I=\begin{pmatrix} -2 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 6 \end{pmatrix}.$$

Identify an angular velocity vector, by specifying its components, that will result in an angular momentum which is parallel to this angular velocity.

5. Evaluate the following integrals：

(a) (14%)

$$\int_0^{2\pi} d\theta \frac{1}{1-2a\sin\theta+a^2} \qquad -1<a<1 \ .$$

(b) (14%)

$$\int_{\infty}^{-\infty} dx \frac{1\mathrm{n}\, x}{1+x^2}.$$

6. (14%) A radioactive isotope I_1 decays into another isotope I_2. I_2 decays, in turn, into I_3 ,and I_3 decays into the stable isotope I_4 . Let $N_j(t)$ be the amount of the isotope I_j at time t,and let c_1, c_2 ,and c_3 be the decay rates. Then ,

$$N_1^{'}=-c_1N_1 \ , \qquad N_2^{'}=c_1N_1-c_2N_2 \ ,$$

$$N_3^{'}=c_2N_2-c_3N_3 \ , \qquad N_4^{'}=c_3N_3 .$$

We assume that $c_1 \neq c_2 \neq c_3$. Suppose one starts at $t=0$ with an amount A of the pure isotope I_1 . How much I_4 is there at time t？