國立彰化師範大學 98 學年度碩士班招生考試試題

系所：物理學系　　組別：甲組　　科目：物理數學

☆☆請在答案紙上作答☆☆　　

1. Let $\phi = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$. Find (a) $\vec{\nabla}\phi$, (b) the directional derivative $\dfrac{d\phi}{ds}$ at $(0,0,1)$ in the direction of $(2\hat{i}+2\hat{j}-\hat{k})$, (c) the direction of maximal decrease at $(0,0,1)$. (15%)

2. Consider the second order differential equation: $\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}-6y=2e^{-3x}$. Find (a) the homogeneous solution, (b) the particular solution, and (c) the general solution. If we define $z(x)=\dfrac{dy}{dx}$, then we can transform the above second order differential equation into a system of first order differential equations, written as

$$\begin{bmatrix} \dfrac{dz}{dx} \\ \dfrac{dy}{dx} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} z \\ y \end{bmatrix} + \begin{bmatrix} 2e^{-3x} \\ 0 \end{bmatrix} .$$

Find (d) the value of $a+b+c+d$. (20%)

3. Consider the matrix $\mathrm{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

Find (a) the eigenvalues of A, and (b) the eigenvectors of A. Now, suppose that A is an $n \times n$ matrix, and that $\mathrm{A}^3 = \mathrm{A}$. (c) What is the possible eigenvalues of A? (15%)

4. Find all the singularities of each of the following functions and describe each of them completely (poles, essential singularities, removable singularities, or branch points?). If the singularities are not branch points, determine the corresponding residues. (18%)

(a) sin (1/*z*);
(b) cos *z*/*z*;
(c) ln (1 + z^2).

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5. Consider a *RC* circuit in which the capacitance *C* and the resistance *R* are connected in series to a bias *v(t)*. Suppose that *v(t)* is a periodic function of period 2π, and

$$v(t)=\begin{cases} 1 & 0 \leqq t \leqq \pi \\ -1 & -\pi < t < 0 \end{cases}$$

(a) Write down the differential equation obeyed by the current *i(t)* in the circuit.
(b) Determine the Fourier series of *v(t)*;
(c) Find *i(t)* as $t \rightarrow +\infty$.

(18%)

6. Consider a harmonic oscillator, with mass *m* and natural frequency ω_0, driven by a force *f(t)* of the form

$$f(t) = I_0\delta(t) .$$

Let the frictional force experienced by the mass be of the form $-2\gamma m dx/dt$ where *x*(*t*) is the displacement of the mass. (14 %)
(a) Write down the differential equation obeyed by *x(t)*.
(b) Suppose that $\gamma > \omega_0$. Determine *x(t)* in terms of the Fourier transform.