# 國立彰化師範大學106學年度碩士班招生考試試題

系所：數學系　　　　科目：線性代數

☆☆請在答案紙上作答☆☆　　　　

**1.** Find all solutions to the following system of linear equations: (15%)

$$\begin{cases} x_1 + 2x_2 + x_3 - x_4 = 2 \\ x_1 + x_2 + x_3 = 3 \\ 3x_1 + 2x_2 + 3x_3 - 2x_4 = 1 \end{cases}$$

**2.** Let $A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$ be a real $2 \times 2$ matrix.

(1) Find an invertible $2 \times 2$ real matrix $P$ such that $P^{-1}AP$ is a diagonal matrix. (15%)

(2) Find an invertible $2 \times 2$ real matrix $P$ such that $P^T AP$ is a diagonal matrix, where $P^T$ is the transpose of $P$. (5%)

**3.** (1) Suppose that $T:\mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $T(1,1) = (1,0,2)$ and $T(2,3) = (1,-1,4)$. What is $T(8,11)$? (7%)

(2) Prove that there exists a linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^3$ such that $T(1,1) = (1,0,2)$ and $T(2,3) = (1,-1,4)$. (8%)

**4.** Suppose $A$ is a $3 \times 4$ matrix and $\operatorname{rank} A = 3$. Determine the nullity of $A, A^T$ and $AA^T$ (nullity of a matrix $M$ is the dimension of its nullspace $\{\mathbf{v} \mid M\mathbf{v} = \mathbf{0}\}$). (15%)

**5.** Determine an orthonormal basis for the plane $P: x - 2y + z = 0$. Find also the projection of $(5, 4, 3)$ on $P$. (15%)

**6.** Let $A$ be an $n \times n$ matrix. Prove that if $A$ is invertible, then, for any linearly independent set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$, $\{A\mathbf{v}_1, A\mathbf{v}_2, \ldots, A\mathbf{v}_k\}$ is linearly independent. Is the converse true? Justify your answer. (20%)