

國立彰化師範大學107學年度碩士班招生考試試題

系所： 統計資訊研究所(選考乙)

科目： 統計學

☆☆請在答案紙上作答☆☆

共 2 頁，第 1 頁

1. Suppose that X is a random variable for which the moment generating function is given by

$$M(t) = \frac{1}{2} + \frac{1}{4}e^{-t} + \frac{1}{4}e^t.$$

- (1) Find $E[X]$ and $\text{Var}(X)$. (10%)
- (2) Compute $E[3X^2 - 2X + 4]$. (5%)
- (3) Compute $\text{Var}\left(\frac{X}{2} - 3\right)$. (5%)

2. Suppose that the joint pdf of times X & Y of first and second arrival of calls to a psychic help line is

$$f(x, y) = 4e^{-2y}, \quad 0 \leq x \leq y < \infty.$$

- (1) Find the marginal pdf of X . (6%)
- (2) Find the conditional pdf of Y given $X = x$. (6%)
- (3) Compute the conditional probability $P\left(Y > 1 \mid X = \frac{1}{2}\right)$. (6%)
- (4) Compute the conditional expected value $E\left(Y \mid X = \frac{1}{2}\right)$. (6%)
- (5) Are X and Y independent? Explain. (6%)

3. Let X_1, \dots, X_n be i.i.d. random samples from a normal distribution, $N(\mu, \sigma^2)$, where μ and σ^2 are unknown. Find the maximum likelihood estimator of

- (1) μ/σ . (10%)
- (2) $P\{X_1 > c\}$ where c is a constant. (10%)

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共 2 頁，第 2 頁

4. Let $\{(X_i, Y_i): i=1, \dots, n\}$ be a sample from a bivariate normal distribution with mean μ_1 and μ_2 , variance 1 and 4, and correlation coefficient 0.5.

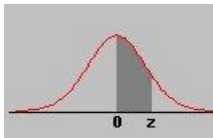
(1) Please find the covariance between sample means $\bar{X} = \sum_i^n X_i/n$ and $\bar{Y} = \sum_i^n Y_i/n$. (10%)

(2) Please find a 95% confidence interval for $2\mu_1 - \mu_2$ by using sample means $\bar{X} = \sum_i^n X_i/n$ and $\bar{Y} = \sum_i^n Y_i/n$. (10%)

(3) We want to test $H_0: 2\mu_1 = \mu_2$ against $H_1: 2\mu_1 > \mu_2$ and would reject H_0 if $2\bar{X} - \bar{Y} > k$.

Please find k for a significant level of 0.05. (10%)

附表：



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990